Constraints on SUSY Gluonic Dipole Interaction from $B \to K\pi$ Decays

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Abstract

In low energy SUSY theories exchange of gluino and squark with left-right mixing can produce a large gluonic dipole interaction. In this paper we study the effects of this interaction on $B \to K\pi$ using QCD improved factorization method. The Standard Model predicts a smaller branching ratio for $B^0 \to \bar{K}^0\pi^0$ than experimental measured one. We find that within the parameter space allowed from $B \to \gamma X_s$ constraint, the SUSY dipole interaction can enhance this branching ratio to agree with the experimental measurement. Combining recent data for all the four $\bar{B}^0 \to K^-\pi^+, \bar{K}^0\pi^0$ and $B^- \to K^-\pi^0, \bar{K}^0\pi^-$ decay modes, we find that the allowed parameter space is reduced significantly compared with that using $B \to X_s\gamma$ data alone. It is found that even with these constraints, the predictions for CP violation in these modes can be dramatically different from those of the SM predictions.

I. INTRODUCTION

There have been considerable experimental and theoretical efforts to understand the properties of B systems. These studies have provided important information about B decays and CP violation. At the quark level the relevant Hamiltonian for B decays in the Standard Model (SM) is well understood. When going beyond the SM, there are new contributions. These new contributions can modify or even completely change the SM predictions [1–3]. In SUSY theories with flavor changing interaction in the squark sector, it is possible to generate large effects on hadronic B decays while their effects on other processes are small [2,3]. In particular exchanges of gluino and squark with left-right mixing can enhance the gluonic dipole interactions of the forms $\bar{q}\sigma^{\mu\nu}G^{\mu\nu}(1\pm\gamma_5)b$ by a large ratio factor of gluino mass $m_{\tilde{g}}$ to b quark mass m_b, $m_{\tilde{g}}/m_b$, compared with the SM prediction. Due to this enhancement factor, even a tiny coefficient for the associated flavor changing squark-gluino-quark interaction, a large gluonic dipole interaction can be generated.

A large gluonic dipole interaction can affect B decays significantly. It may help to explain the large branching ratios observed for $B \to X_s\eta'$, although theoretical understanding is poor [4,5]. It can also change theoretical predictions for other charmless hadronic B decays, such as $B \to K\pi, \pi\pi, \phi K$ [5,6]. Using the recently measured branching ratios for $B \to K\pi, \pi\pi$, one may be able to constraint the allowed parameter space which can generate a large gluonic dipole interactions and to provide interesting information about models beyond the SM. In SUSY models the same left-right squark mixing parameters can also generate a photonic dipole interaction which can induce $B \to X_s\gamma$ and $B \to X_d\gamma$. At present $B \to X_s\gamma$ has been observed, but not $B \to X_d\gamma$.

In this paper we study in detail constraints on SUSY gluonic dipole interaction using data from $\bar{B}^0 \to K^-\pi^+, \bar{K}^0\pi^0$ and $B^- \to K^-\pi^0, \bar{K}^0\pi^-$ and also the measured branching ratio of $B \to X_s\gamma$. Previous studies of gluonic dipole interactions on B decays were based on naive factorization approximation. Here we will use QCD improved factorization method [7,8] which improves the analysis on several aspects, such as the number of color, the gluon

virtuality, the renormalization scale, scheme dependencies and etc. We find that the recent data on $B \to K\pi$ decays can reduce, significantly, the allowed parameter space compared with using $B \to X_s\gamma$ data alone, while still allow large deviations from the SM predictions for branching ratios and CP asymmetries in these decay modes.

The paper is arranged as follows. In section II, we discuss gluonic dipole interactions in low energy SUSY model with left-right squark mixing, and obtain the decay amplitudes for $B \to K\pi$ decays using QCD improved factorization. In section III, we first update constraints on the SUSY gluonic dipole interactions from $B \to X_s\gamma$, and then study constraints from $B \to K\pi$ decays. In section IV, we study CP rate asymmetry in $B \to K\pi$. And in section V, we draw our conclusions.

II. SUSY GLUONIC DIPOLE CONTRIBUTIONS TO $B \to K\pi$

In the SM the effective Hamiltonian for charmless photonic and hadronic B decays with $\Delta S = 1$ at the quark level is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^*(c_1O_1 + c_2O_2 + \sum_{i=3}^{12} c_iO_i) + V_{cb}V_{cs}^* \sum_{i=3}^{12} c_iO_i \right\}. \qquad (1)$$

Here O_i are quark, gluon and photon operators and are given by

$$\begin{aligned}
&O_1 = (\bar{s}_i u_i)_{V-A}(\bar{u}_j b_j)_{V-A}, \quad O_2 = (\bar{s}_i u_j)_{V-A}(\bar{u}_i b_j)_{V-A},\\
&O_{3(5)} = (\bar{s}_i b_i)_{V-A} \sum_{q'} (\bar{q}'_j q'_j)_{V-(+)A}, \quad O_{4(6)} = (\bar{s}_i b_j)_{V-A} \sum_{q'} (\bar{q}'_j q'_i)_{V-(+)A},\\
&O_{7(9)} = \frac{3}{2}(\bar{s}_i b_i)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_j q'_j)_{V+(-)A}, \quad O_{8(10)} = \frac{3}{2}(\bar{s}_i b_j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_j q'_i)_{V+(-)A},\\
&O_{11} = \frac{g_s}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} G^a_{\mu\nu} T_a^{ij} (1+\gamma_5) b_j, \quad O_{12} = \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} F_{\mu\nu} (1+\gamma_5) b_i,
\end{aligned} \qquad (2)$$

where $(V \pm A)(V \pm A) = \gamma^\mu(1 \pm \gamma_5)\gamma_\mu(1 \pm \gamma_5)$, $q' = u, d, s, c, b$, $e_{q'}$ is the electric charge number of the q' quark, T_a is the color SU(3) generator normalized as $Tr(T^aT^b) = \delta^{ab}/2$, i and j are color indices, and $G_{\mu\nu}$ and $F_{\mu\nu}$ are the gluon and photon field strengths.

The Wilson coefficients c_i have been calculated in different schemes [9]. In this paper we will use consistently the NDR scheme. The values of c_i at $\mu \approx m_b$ with QCD corrections at

NLO are given by [7]

$$c_1 = 1.081,\ \ c_2 = -0.190,\ \ c_3 = 0.014,\ \ c_4 = -0.036,\ \ c_5 = 0.009,\ \ c_6 = -0.042,$$
$$c_7 = -0.011\alpha_{em},\ \ c_8 = 0.060\alpha_{em},\ \ c_9 = -1.254\alpha_{em},\ \ c_{10} = 0.223\alpha_{em}. \tag{3}$$

And at LO are given by [7]

$$c_1 = 1.117,\ \ c_2 = -0.268,\ \ c_3 = 0.012,\ \ c_4 = -0.027,\ \ c_5 = 0.008,\ \ c_6 = -0.034,$$
$$c_7 = -0.001\alpha_{em},\ \ c_8 = 0.029\alpha_{em},\ \ c_9 = -1.276\alpha_{em},\ \ c_{10} = 0.288\alpha_{em},$$
$$c_{11} = -0.151,\ \ c_{12} = -0.318. \tag{4}$$

Here $\alpha_{em} = 1/129$ is the electromagnetic fine structure constant. We use LO Wilson coefficients with the matrix elements enter the decay amplitudes at next-to-leading order.

In SUSY models, exchanges of gluino and squark with left-right squark mixing, can generate a large contribution to $c_{11,12}$ at one loop level. In general these contributions can generate a gluonic dipole interaction with the same chirality as the SM one, as well as with an opposite chirality as the SM one, that is, an interaction similar to $O_{11,12}$ but with $1+\gamma_5$ replaced by $1-\gamma_5$. It is difficult to carry out an analysis in the full parameter space. We will first consider the new contributions with the same chirality as the SM one setting the opposite chirality one to zero, and then the opposite case.

The effective Wilson coefficient for $c_{11,12}^{susy}$ obtained in the mass insertion approximation is given by, for the case with the same chirality as the SM ones [10],

$$\begin{aligned}
c_{11}^{susy}(m_{\tilde{g}}) &= \frac{\sqrt{2}\pi\alpha_s(m_{\tilde{g}})}{G_F m_{\tilde{g}}^2}\frac{\delta_{LR}^{bs}}{V_{tb}V_{ts}^*}\frac{m_{\tilde{g}}}{m_b}G_0(x_{gq}),\\
c_{12}^{susy}(m_{\tilde{g}}) &= \frac{\sqrt{2}\pi\alpha_s(m_{\tilde{g}})}{G_F m_{\tilde{g}}^2}\frac{\delta_{LR}^{bs}}{V_{tb}V_{ts}^*}\frac{m_{\tilde{g}}}{m_b}F_0(x_{gq}),\\
G_0(x) &= \frac{x}{3(1-x)^4}[22 - 20x - 2x^2 + 16x\ln(x) - x^2\ln(x) + 9\ln(x)],\\
F_0(x) &= -\frac{4x}{9(1-x)^4}[1 + 4x - 5x^2 + 4x\ln(x) + 2x^2\ln(x)],
\end{aligned} \tag{5}$$

where δ_{LR}^{bs} is the mixing parameter of left and right squarks, $x_{gq} = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ is the ratio of gluino and squark mass.

The coefficients $c^{susy}_{11,12}(m_{\tilde{g}})$ at m_b are given by [10]

$$c^{susy}_{11}(\mu) = \eta c^{susy}_{11}(m_{\tilde{g}}), \;\; c^{susy}_{12}(\mu) = \eta^2 c^{susy}_{12}(m_{\tilde{g}}) + \frac{8}{3}(\eta - \eta^2) c^{susy}_{11}(m_{\tilde{g}}), \tag{6}$$

where $\eta = (\alpha_s(m_{\tilde{g}})/\alpha_s(m_t))^{2/21}(\alpha_s(m_t)/\alpha_s(mb))^{2/23}$.

In the SM, $c_{11,12}$ are proportional to m_b/m^2_W. From the above expressions, it is clear that the SUSY contributions are proportional to $1/m_{\tilde{g}}$. If $m_{\tilde{g}}$ is of order a few hundred GeV, there is an enhancement factor of $m_{\tilde{g}}/m_b(m^2_W/m^2_{\tilde{g}})$ for the SUSY gluonic dipole interaction. In this case even a small δ^{bs}_{LR} can have a large effect on B decays.

To obtain $c^{susy}_{11,12}$ for opposite chirality case, one just adds in two more operators similar to $O_{11,12}$ but with $1+\gamma_5$ replaced by $1-\gamma_5$ and the Left-Right mixing parameter δ^{bs}_{LR} replaced by the Right-Left mixing parameter δ^{bs}_{RL}.

We follow Ref. [7] to obtain the $B \to K\pi$ decay amplitudes. They are given by

$$\begin{aligned}
A(\bar{B}^0 \to \bar{K}^0\pi^0) &= \frac{G_F}{2} i f_\pi (m^2_B - m^2_K) F_0^{B\to K}(m^2_\pi)[V_{ub}V^*_{us}(a_2 + \frac{3}{2}(-a_7 + a_9)) + V_{cb}V^*_{cs}\frac{3}{2}(-a_7 + a_9)] \\
&+ \frac{G_F}{2} i f_K (m^2_B - m^2_\pi) F_0^{B\to\pi}(m^2_K)[V_{ub}V^*_{us}(-a^u_4 + \frac{1}{2}a^u_{10} - R_K(a^u_6 - \frac{1}{2}a^u_8)) \\
&+ V_{cb}V^*_{cs}(-a^c_4 + \frac{1}{2}a^c_{10} - R_K(a^c_6 - \frac{1}{2}a^c_8)] - \frac{G_F}{2} i f_B f_\pi f_K (V_{ub}V^*_{us} + V_{cb}V^*_{cs})(b_3 - \frac{1}{2}b^{EW}_3),
\end{aligned}$$

$$\begin{aligned}
A(\bar{B}^0 \to K^-\pi^+) &= \frac{G_F}{2} i f_\pi (m^2_B - m^2_K) F_0^{B\to K}(m^2_\pi)[V_{ub}V^*_{us}(a_1 + a^u_4 + a^u_{10} + R_K(a^u_6 + a^u_8)) \\
&+ V_{cb}V^*_{cs}(a^c_4 + a^c_{10} + R_K(a^c_6 + a^c_8)] + \frac{G_F}{\sqrt{2}} i f_B f_\pi f_K (V_{ub}V^*_{us} + V_{cb}V^*_{cs})(b_3 - \frac{1}{2}b^{EW}_3),
\end{aligned}$$

$$\begin{aligned}
A(B^- \to K^-\pi^0) &= \frac{G_F}{2} i f_K (m^2_B - m^2_\pi) F_0^{B\to\pi}(m^2_\pi)[V_{ub}V^*_{us}(a_2 + \frac{3}{2}(-a_7 + a_9)) + V_{cb}V^*_{cs}\frac{3}{2}(-a_7 + a_9))] \\
&+ \frac{G_F}{2} i f_K (m^2_B - m^2_\pi) F_0^{B\to\pi}(m^2_K)[V_{ub}V^*_{us}(a_2 + a^u_4 + a^u_{10} + R_K(a^u_6 + a^u_8)) \\
&+ V_{cb}V^*_{cs}(a^c_4 + R_K(a^c_6 + a^c_8 + a^c_{8a}) + a^c_{10})] + \frac{G_F}{\sqrt{2}} i f_B f_\pi f_K [V_{ub}V^*_{us}(b_2 + b_3 + b^{EW}_3) + V_{cb}V^*_{cs}(b_3 + b^{EW}_3)],
\end{aligned}$$

$$\begin{aligned}
A(B^- \to \bar{K}^0\pi^-) &= \frac{G_F}{2} i f_K (m^2_B - m^2_\pi) F_o^{B\to\pi}(m^2_K)[V_{ub}V^*_{us}(a^u_4 - \frac{1}{2}a^u_{10} + R_K(a^u_6 - \frac{1}{2}a^u_8)) \\
&+ V_{cb}V^*_{cs}(a^c_4 + R_K(a^c_6 - \frac{1}{2}a^c_8) - \frac{1}{2}a^c 10)] + \frac{G_F}{\sqrt{2}} i f_B f_\pi f_K [V_{ub}V^*_{us}(b_2 + b_3 + b^{EW}_3) + V_{cb}V^*_{cs}(b_3 + b^{EW}_3)].
\end{aligned} \tag{7}$$

Here $R_K = 2m_K^2/m_s m_b$. a_i and b_i coefficients are related to the Wilson coefficients. In the above we have neglected small contributions from O_{12}. Including the lowest α_s order corrections, a_i^q are given by

$$
\begin{aligned}
a_1 &= c_1 + \frac{c_2}{N}\left[1 + \frac{C_F\alpha_s}{4\pi}V_K\right] + \frac{c_2}{N_c}\frac{C_F\pi\alpha_s}{N_c}H_{K\pi},\\
a_2 &= c_2 + \frac{c_1}{N}\left[1 + \frac{C_F\alpha_s}{4\pi}V_\pi\right] + \frac{c_1}{N_c}\frac{C_F\pi\alpha_s}{N_c}H_{\pi K},\\
a_4^p &= c_4 + \frac{c_3}{N}\left[1 + \frac{C_F\alpha_s}{4\pi}V_K\right] + \frac{C_F\alpha_s}{4\pi}\frac{P^p_{K,2}}{N_c} + \frac{c_3}{N_c}\frac{C_F\pi\alpha_s}{N_c}H_{K\pi},\\
a_6^p &= c_6 + \frac{c_5}{N}\left[1 - 6\cdot\frac{C_F\alpha_s}{4\pi}\right] + \frac{C_F\alpha_s}{4\pi}\frac{P^p_{K,3}}{N_c},\\
a_7 &= c_7 + \frac{c_8}{N}\left[1 + \frac{C_F\alpha_s}{4\pi}(-V'_\pi)\right] + \frac{c_8}{N_c}\frac{C_F\pi\alpha_s}{N_c}(-H'_{\pi K}),\\
a_8^p &= c_8 + \frac{c_7}{N}\left[1 - 6\cdot\frac{C_F\alpha_s}{4\pi}\right] + \frac{\alpha_{em}}{9\pi}\frac{P^{p,EW}_{K,3}}{N_c},\\
a_9 &= c_9 + \frac{c_{10}}{N}\left[1 + \frac{C_F\alpha_s}{4\pi}V_\pi\right] + \frac{c_{10}}{N_c}\frac{C_F\pi\alpha_s}{N_c}H_{\pi K},\\
a_{10}^p &= c_{10} + \frac{c_9}{N}\left[1 + \frac{C_F\alpha_s}{4\pi}V_K\right] + \frac{\alpha_{em}}{9\pi}\frac{P^{p,EW}_{K,2}}{N_c} + \frac{c_9}{N_c}\frac{C_F\pi\alpha_s}{N_c}H_{K\pi},
\end{aligned}
\tag{8}
$$

where $C_F = (N_c^2-1)/(2N_c)$, and $N_c = 3$. The quantities $V_M^{(\prime)}$, $H^{(\prime)}_{M_2M_1}$, $P^p_{K,2}$, $P^p_{K,3}$, $P^{p,EW}_{K,2}$ and $P^{p,EW}_{K,3}$ are hadronic parameters that contain all nonperturbative dynamics.

The vertex corrections $V_M^{(\prime)}$(M=π,K) are give by

$$
\begin{aligned}
V_M &= 12\ln\frac{m_b}{\mu} - 18 + \int_0^1 dx g(x)\phi_M(x),\\
V'_M &= 12\ln\frac{m_b}{\mu} - 18 + \int_0^1 dx g(1-x)\phi_M(x),\\
g(x) &= 3\frac{1-2x}{1-x}\ln x - 3i\pi\\
&\quad + \left[2\mathrm{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3+2i\pi) - (x\leftrightarrow 1-x)\right],
\end{aligned}
\tag{9}
$$

where $\mathrm{Li}_2(x)$ is the dilogarithm. $\phi_M(x)$ is the leading-twist light cone distribution amplitude. This distribution amplitude can be expansion in Gegenbauer ploynomials. We truncate this expansion at $n=2$.

$$
\phi_M(x,\mu) = 6x(1-x)\left[1 + \alpha_1^M(\mu)C_1^{(3/2)}(2x-1) + \alpha_2^M(\mu)C_2^{(3/2)(2x-1)}\right],
\tag{10}
$$

where $C_1^{(3/2)}(u) = 3u$ and $C_2^{(3/2)}(u) = \frac{3}{2}(5u^2-1)$. The distribution amplitude parameters $\alpha_{1,2}^M$ for $M = K, \pi$ are: $\alpha_1^K = 0.3$, $\alpha_2^K = 0.1$, $\alpha_1^\pi = 1$ and $\alpha_2^\pi = 0.1$.

The penguin contributions $P_{K,2}^p$, $P_{K,3}^p$, $P_{K,2}^{p,EW}$ and $P_{K,3}^{p,EW}$ are given by

$$\begin{aligned}
P_{K,2}^p &= c_1\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-G_K(s_p)\right]+c_3\left[\frac{8}{3}\ln\frac{m_b}{\mu}+\frac{4}{3}-G_K(0)-G_K(1)\right]\\
&\quad+(c_4+c_6)\left[\frac{4n_f}{3}\ln\frac{m_b}{\mu}-(n_f-2)G_K(0)-G_K(s_c)-G_K(1)\right]\\
&\quad-2c_{11}\int_0^1\frac{dx}{1-x}\phi_K(x),\\
P_{K,2}^{p,EW} &= (c_1+N_c c_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{3}{2}-G_K(s_p)\right]-2c_{12}\int_0^1\frac{dx}{1-x}\phi_K(x),\\
P_{K,3}^p &= c_1\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{2}{3}-\hat{G}_K(s_p)\right]+c_3\left[\frac{8}{3}\ln\frac{m_b}{\mu}+\frac{4}{3}-\hat{G}_K(0)-\hat{G}_K(1)\right]\\
&\quad+(c_4+c_6)\left[\frac{4n_f}{3}\ln\frac{m_b}{\mu}-(n_f-2)\hat{G}_K(0)-\hat{G}_K(s_c)-\hat{G}_K(1)\right]-2c_{11},\\
P_{K,3}^{p,EW} &= (c_1+N_c c_2)\left[\frac{4}{3}\ln\frac{m_b}{\mu}+\frac{3}{2}-\hat{G}_K(s_p)\right]-3c_{12},
\end{aligned}\tag{11}$$

with

$$\begin{aligned}
G_K(s) &= \int_0^1 dx G(s-i\epsilon, 1-x)\phi_K(x),\\
\hat{G}_K(s) &= \int_0^1 dx G(s-i\epsilon, 1-x)\phi_p^K(x),\\
G(s,x) &= -4\int_0^1 du u(1-u)\ln[s-u(1-u)x],
\end{aligned}\tag{12}$$

where $\phi_p^K(x) = 1$.

The hard-scattering contributions $H_{\pi K}$, $H'_{\pi K}$, and $H_{K\pi}$ contain many poorly know parameters,we follow [7] use $H_{\pi K} = 0.99$ and

$$H'_{\pi K} = H_{\pi K}, \quad H_{K\pi} = R_{\pi K} H_{\pi K}, \tag{13}$$

where

$$R_{\pi K} \simeq \frac{F_0^{B\to K}(0) f_\pi}{F_0^{B\to\pi}(0) f_K}. \tag{14}$$

All scale-dependent quantities for hard spectator contributions are evaluated at $\mu_h = \sqrt{\Lambda_h m_b}$ with $\Lambda_h = 0.5\text{GeV}$.

The annihilation coefficients are

$$\begin{aligned}
b_2 &= \frac{C_F}{N_c^2} c_2 A_1^i, \quad b_3 = \frac{C_F}{N_c^2}[c_3 A_1^i + c_5(A_3^i + A_3^f) + N_c c_6 A_3^f], \\
b_3^{EW} &= \frac{C_F}{N_c^2}[c_9 A_1^i + c_7(A_3^i + A_3^f) + N_c c_8 A_3^f], \\
b_4^{EW} &= \frac{C_F}{N_c^2}[c_{10} A_1^i + c_8 A_2^i].
\end{aligned} \tag{15}$$

with

$$\begin{aligned}
A_1^i &= A_2^i = \pi\alpha_s \left[18\left(X_A - 4 + \frac{\pi^2}{3}\right) + 2r_\chi^2 X_A^2\right], \\
A_3^f &= 12\pi\alpha_s r_\chi^2 (2X_A^2 - X_A),
\end{aligned} \tag{16}$$

where $r_\chi = 2\mu_h/m_b$, $X_A = \int_0^1 dy/y$ parameterizes the divergent endpoint integrals, We use $X_A = \ln(m_B/\Lambda_h)$. All scale dependent quantities for annihilation contributions are evaluated at μ_h.

In the numerical evaluation of these expressions we follow Ref. [7] consistently drop high-order terms in the products of the Wilson coefficients with the next-to-leading order corrections. I.e., we evaluate all the corrections beyond naive factorization with leading order Wilson coefficients.

For the case where the SUSY contributions have the same chirality as the SM one, $c_{11} = c_{11}^{SM} + c_{11}^{susy}$. For the opposite chirality case, one needs to change c_{11} to $c_{11}^{SM} - c_{11}^{susy}$ due to the replacement of $1 + \gamma_5$ by $1 - \gamma_5$.

III. CONSTRAINTS ON C_{11} FROM $B \to \gamma X_S$ AND $B \to K\pi$

A. Constraints from $B \to X_s\gamma$

We now study the constraints on gluonic dipole interaction using $B \to X_s\gamma$. To this end we define $r_\gamma = |c_{12}^{susy}(m_b)/c_{12}^{SM}(m_b)|$ and $r_g = |c_{11}^{susy}(m_b)/c_{11}^{SM}(m_b)|$. $c_{11,12}^{SM}(m_b)$ denote SM contributions, which are given in the Eq.(4). Using Eq.(6) and replacing $c_{11,12}^{susy}$ with Eq.(5), we obtain

$$\frac{r_g}{r_\gamma} = [\eta \frac{F_0(x_{gq})}{G_0(x_{gq})} + \frac{8}{3}(1-\eta)]^{-1} \frac{c_{12}^{SM}}{c_{11}^{SM}} \ . \tag{17}$$

r_γ/r_g is not sensitive to gluino mass, but strongly depends on x_{gq}. In Fig.(1) we show r_g/r_γ as a function of x_{gq} for different gluino masses. Note that both r_g and r_γ have a common CP violating phase δ which is equal to the phase difference of the SM phase and the SUSY phase of δ_{LR}^{sb}.

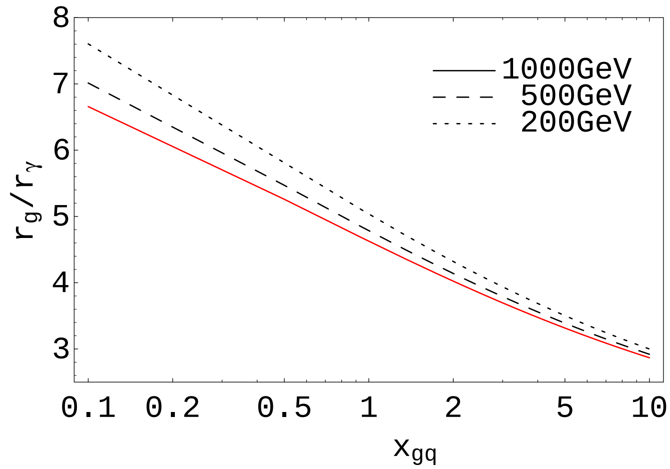

FIG. 1. Ratio of r_g/r_γ vs. x_{gq}. The solid, dashed and dotted lines correspond to gluino mass of 1000 GeV, 500 GeV and 200 GeV, respectively.

For a given r_γ, r_g is known as a function of x_{gq}. Therefore constraints on r_γ from $B \to X_s\gamma$ can be translated into constraint on r_g. The constraints on r_γ is given by [3,11]

$$Br(B \to X_s\gamma) = Br(B \to X e\bar{\nu}_e) \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{em}}{\pi g(m_c/m_b)\eta} |c_{12}^{eff}(m_b)|^2 \ , \tag{18}$$

where $c_{12}^{eff}(M_b) = c_{12}^{SM}(m_b)(1 + r_\gamma e^{i\delta})$, $g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 lnz$, and $\eta = 1 - 2f(r,0,0)\alpha_s(m_b)/3\pi$ with $f(r,0,0) = 2.41$ [11]. Here we have used the leading order result. The use of next-leading order result [12] will slightly change the results (at the level of 10%), but will not change the main conclusions.

For the opposite chirality case, one needs to replace $|c_{12}^{eff}|^2 = |c_{12}^{SM}(1 + r_\gamma e^{i\delta})|^2$ in the above by $|c_{12}^{eff}|^2 = |c_{12}^{SM}|^2(1 + r_\gamma^2)$ because the SM and SUSY contributions have different chiralities. Note that in this case $r_{\gamma,g}$ also have a common phase equal to the phase difference of SM and SUSY due to δ_{RL}^{sb} which will be also indicated by δ.

In the numerical analysis we will use $Br(B \to Xe\bar{\nu}_e) = 10.4\%$, $m_b = 4.2GeV$, $m_c = 1.3GeV$, and $|V_{ts}^* V_{tb}|^2/|V_{cb}|^2 = 0.95$. Using the experimental allowed range $Br(B \to X_s\gamma) = (2.96 \pm 0.35) \times 10^{-4}$ averaged from CLEO, ALEPH, and Belle [13], one can obtain the allowed region for r_γ by Eqs.(13) and (14). The 95% C.L. region of r_γ is shown in Fig.(2). Combining information from Figs.(1) and (2), we obtain the allowed region for r_g in Fig.(3). We see that c_{11}^{susy} can be considerably larger than the SM. We note that the bounds of r_γ and r_g in the opposite chirality case can be up twice depend on the choose of the parameters m_c/m_b and c_{12}. It is, however, not change our results too much in the later discussion.

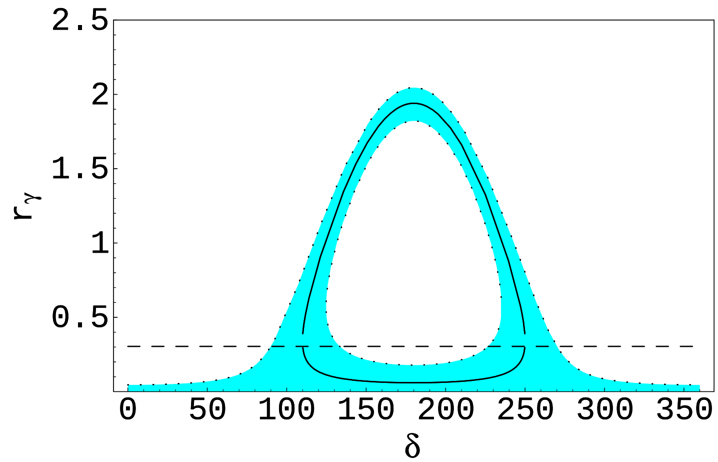

FIG. 2. Constraint on r_γ from $B \to X_s\gamma$. For the case with the same chirality as the SM one, the 95% C.L. allowed region is indicated by the shaded region. For the opposite chirality case, the 95% C.L. allowed region is below the dashed line.

B. Constraints from $B \to K\pi$

It is clear from discussions in the previous section $B \to X_s\gamma$ can constrain c_{11}^{susy}, but still allow large deviations from the SM prediction. Within the allowed regions for r_g, rare $B \to K\pi$ decays can be dramatically different from the SM predictions. Therefore using experimental data on $B \to K\pi$ given in Table I [14], one can further constrain the allowed regions for r_g.

The branching ratio for $B \to K\pi$ with SUSY contribution to c_{11} can be easily obtained

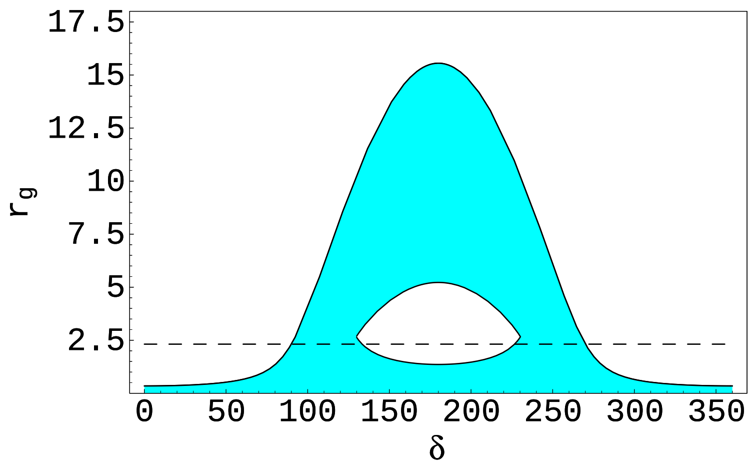

FIG. 3. Constraint of r_g in 95%C.L., with $200 < m_{\tilde{g}} < 1000$, $0.1 < x_{gq} < 10$. The shaded region and the region below the dashed line are the allowed regions with the 95% C.L. for the case with the same and opposite chiralities, respectively.

by using $c_{11} = c_{11}^{SM}(1 + r_g e^{i\delta})$ in Eq.(8) for the SUSY contributions with the same chirality as the SM one, and by using $c_{11} = c_{11}^{SM}(1 - r_g e^{i\delta})$ with opposite chirality case. To finally obtain the branching ratios, we need to know the form factors, $F_0^{B\to P}$, the quark masses, the parameters of distribution amplitudes, the CKM parameters V_{ub}, V_{cb} and the phase γ. There are several theoretical calculations for the form factors, we will use $F_0^{B\to\pi} = 0.28$ and $R_{\pi K} = 0.9$ given in Ref. [7]. The quark masses $m_b(m_b) = 4.2GeV$, $m_c(m_b) = 1.3GeV$, $m_s(2GeV) = 110MeV$ and $(m_u + m_d)(2GeV) = 9.1MeV$ will be used for illustration. For the magnitudes of the CKM parameters we will use $|V_{cb}| = 0.0402$ and $|V_{ub}/V_{cb}| = 0.090$ and treat γ as a free parameter.

To see how the SUSY gluonic dipole interactions can change the SM predictions in detail, we first study some special cases for O_{11} with the same chirality as the SM one, and then consider the combined constraints. For the special cases we take r_γ determined using the central value for $B \to X_s\gamma$, and take the corresponding r_g with $x_{gq} = 10$ to minimize the effects. For the CP violating phase we consider three scenarios: a) $r_g e^{i\delta}$ is real and the phase γ is the only CP violating phase; b) the phase γ is set to zero and the new contribution has a phase δ which can vary from 0 to 2π; And c) the phase γ is fixed at the current best fit value $\gamma = 66^\circ$ [16] in the SM and let the phase δ to vary from 0 to 2π.

Branching ratio		data	Average
$Br(B \to K^+\pi^-)$	CLEO	$17.2^{+2.5}_{-2.4} \pm 1.2$	17.3 ± 1.5
	Belle	$19.3^{+3.4+1.5}_{-3.2-0.6}$	
	BaBar	$16.7 \pm 1.6 \pm 1.3$	
$BR(B \to K^-\pi^0)$	CLEO	$11.6^{+3.0+1.4}_{-2.7-1.3}$	12.1 ± 1.7
	Belle	$16.3^{+3.5+1.6}_{-3.3-1.8}$	
	BaBar	$10.8^{+2.1}_{-1.9} \pm 1.0$	
$Br(B \to \bar{K}^0\pi^-)$	CLEO	$18.2^{+4.6}_{-4.0} \pm 1.6$	17.3 ± 2.7
	Belle	$13.7^{+5.7+1.9}_{-4.8-1.8}$	
	BaBar	$18.2^{+3.3}_{-3.0} \pm 2.0$	
$Br(B \to K^0\pi^0)$	CLEO	$14.6^{+5.9+2.4}_{-5.1-3.3}$	10.4 ± 2.7
	Belle	$16.0^{+7.2+2.5}_{-5.9-2.7}$	
	BaBar	$8.2^{+3.1}_{-2.7} \pm 1.2$	

TABLE I. Experimental data of $B \to K\pi$ decays from CLEO, Belle and BaBar [14]

The results are shown in Fig.(4). For comparison, we first show the SM predictions for the four $B \to K\pi$ decays in Fig.(4.i). The γ phase seem to be allowed only in the small regions around $100° \sim 109°$ and $251° \sim 260°$, which are different from the globe fitting [16]. Of course this situation can be different when different parameters are used [7]. we will take the value obtained here for illustration.

The situation can be dramatically changed if SUSY gluonic dipole interactions are included. In case a), there are two regions of solutions of $r_g e^{i\delta}$, one is small negative and the other is negative with large magnitude. The small negative solution $r_g e^{i\delta}$ is too small to influence $B \to K\pi$ decay. For the large negative $r_g e^{i\delta}$ we find that the gluino contributions is 5.6 times larger then the SM one, but have opposite sign to the SM one. In this case, all $B \to K\pi$ decays are too large as can be seen from Fig.(4.ii). If a smaller x_{gq} is used, the situation is worse. SUSY contribution with no phase ($\delta = 0$) is not favored.

In case b), when the phase δ is small, the branching ratios are almost the same as those in the SM which increase with the phase δ to maximal at $\delta = 180°$. There is a large gap between $B \to K^0\pi^-$ and $B \to K^+\pi^-$ shown in Fig.(4.iii). $B \to \bar{K}^0\pi^0$ and $B \to K^-\pi^0$ are overlap. The regions with branching ratios for $B \to K\pi$ to be within experimental 2σ ranges are located between $117° \sim 120°$ and $240° \sim 243°$.

In case c), the four branching ratios are shown in Fig.(4.iv). In this case it is, again, possible to make all decays into experimental 2σ ranges. The overlap regions for δ are located between $109° \sim 121°$ and $243° \sim 261°$.

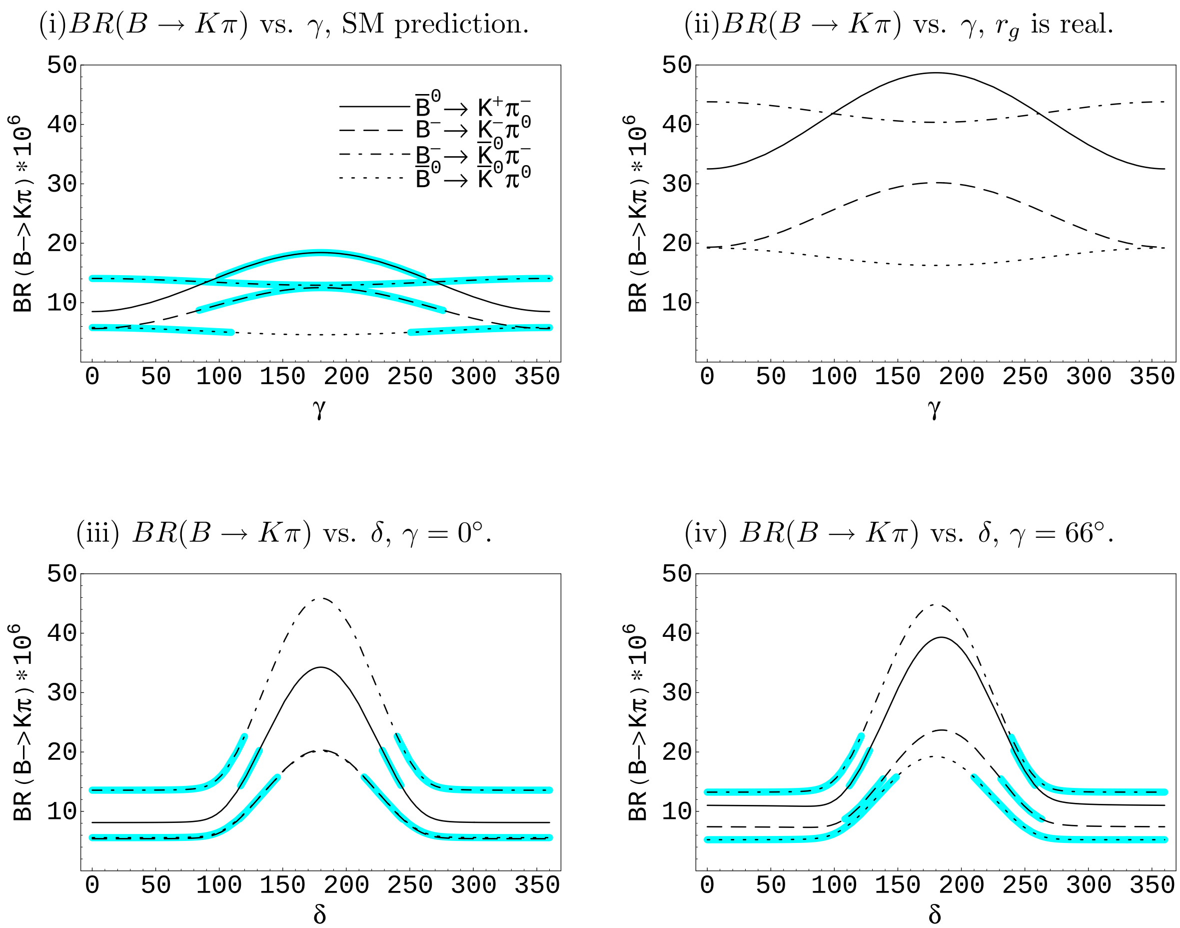

FIG. 4. Branching ratios of $B \to K\pi$. The line segments with shade indicate the branching ratios which are within the experimental 2σ regions.

From the above discussions, we see that SUSY gluonic dipole interaction can improve

agreement of theoretical predictions and experimental data. Both CP violating phases γ and δ can affect the branching ratios significantly.

Since the gluonic dipole interaction can have significant influence on $B \to K\pi$ branching ratios, $B \to K\pi$ decays can, therefore, also be used to constrain SUSY gluonic dipole interaction. To obtain the restricted regions, we scan the allowed region of r_g obtained earlier from $B \to X_s\gamma$ and the SUSY parameter space $0.1 < x_{gq} < 10$, 200 GeV $< m_{\tilde{g}} <$ 1000 GeV. The allowed regions for $Br(B \to K\pi)$ within 2σ of experimental values are shown in Fig.(5). In obtaining the allowed regions in Fig.(5), we treated γ as a free parameter varying in the range $0^\circ \sim 360^\circ$. Figs.(5.a-d) show the allowed regions from each $B \to K\pi$ decay. All four decays constrain r_g to be less than 7.2. $\bar{B}^0 \to \bar{K}^0\pi^-$ provides the most powerful constraint on allowed region for r_g. The regions with $r_g > 4.4$ are ruled out at 95% C.L..

In Fig.(5.e) we use all four $B \to K\pi$ decays to constrain r_g with γ varying from 0° to 360°. r_g is constrained to be less than 4.4. δ between $0^\circ \sim 80^\circ$ and $280^\circ \sim 360^\circ$ are allowed with small r_g disfavored due to the reason that it make the branching ratio of $B \to K\pi$ smaller. We also show the constraint with $42^\circ < \gamma < 87^\circ$, which is the 95% C.L. of γ from the fit of unitarity triangle in the SM [16] in Fig.(5.f). In this case we see that the allowed regions of r_g and δ are further reduced, $0.3 < r_g < 4.3$ and δ is between $100^\circ \sim 274^\circ$. Non-zero r_g and δ fit data better than SM.

The constraints on the SUSY gluonic dipole interactions with opposite chirality to the SM one can be easily obtained by using $c_{11} = c_{11}^{SM}(1 - r_g e^{i\delta})$ in Eq.(8). The combined allowed regions on r_g, under the same conditions as for the previous discussions, are shown in Fig.(6). Data from $B \to K\pi$, again, can further constrain the allowed parameter space compared with constraint from $B \to X_s\gamma$ alone. The region between $100^\circ < \delta < 260^\circ$ is not favored.

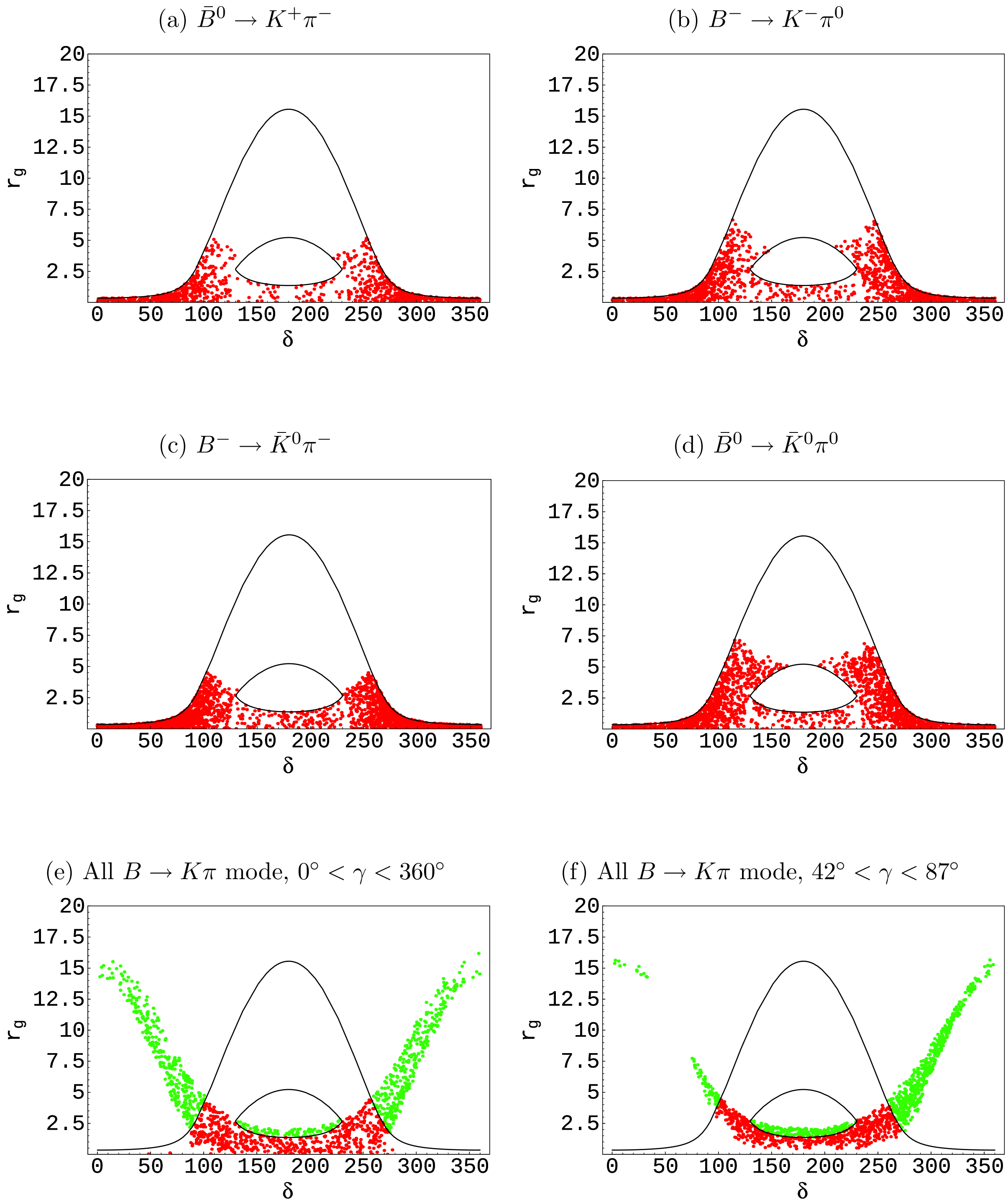

FIG. 5. Constraints on r_g using $B \to X_s\gamma$ and $B \to K\pi$ decays. The solid lines are the boundaries of the constraint from $B \to X_s\gamma$ and the dotted regions are the allowed regions by data from $B \to K\pi$ at 2σ level.

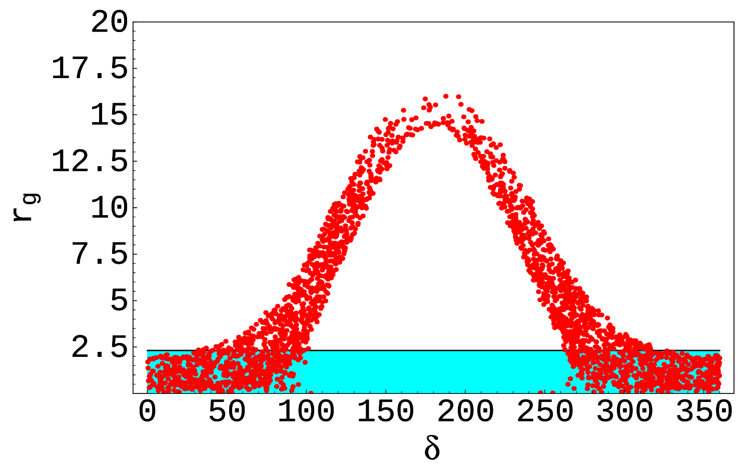

FIG. 6. The allowed regions for SUSY gluonic dipole interaction with opposite chirality as the SM one. The dotted regions are the allowed regions from $B \to K\pi$ constraints, and the region below the dashed line is the upper bound from $B \to X_s\gamma$ constraint.

IV. CP VIOLATION IN $B \to K\pi$ WITH SUSY CONTRIBUTIONS

From previous discussions, it is clear that SUSY gluonic dipole interactions can affect the branching ratios of $B \to K\pi$ significantly. It is interesting to note that large new CP violating phase δ is still allowed which may affect CP violation in these decays. In this section we study CP violating rate asymmetry

$$A_{asy} = \frac{\bar{\Gamma} - \Gamma}{\bar{\Gamma} + \Gamma}, \tag{19}$$

with SUSY gluonic dipole interactions.

The allowed CP asymmetry A_{asy} for each of the $B \to K\pi$ decay is obtained by using the allowed regions of parameter space constrained from the previous section. The results for SUSY gluonic dipole interactions with the same chirality as the SM one and opposite one are shown in Fig. (7) and Fig. (8), respectively. The reference SM predictions as a function of γ are shown as solid lines in Fig.(7). When SUSY gluonic dipole contributions are included, the asymmetries can be dramatically different because r_g and δ both can be large. The scattered dots in Fig. (7) above and bellow the SM predictions correspond to the regions on $\delta > 180°$ and $\delta < 180°$ in Fig.(5e), respectively. We see clearly that the predictions can be much larger than the SM predictions. For example, with $r_g = 3$, $\delta = 120°$

and $\gamma = 66°$, the branching ratios for $K^+\pi^-$, $K^-\pi^0$, $\bar{K}^0\pi^-$, and $\bar{K}^0\pi^0$, are predicted to be $(16.8,\ 10.7,\ 21.8,\ 9.2) \times 10^{-6}$ which are within the 2σ region of data, and the asymmetries are predicted to be -0.04, 0.00, -0.06, -0.10, respectively. With $r_g = 3$, $\delta = 250°$ and $\gamma = 66°$, the branching ratios are $(19.0,\ 12.1,\ 20.0,\ 8.1) \times 10^{-6}$, and the asymmetries are 0.11, 0.12, 0.09, 0.07, respectively.

For SUSY gluonic dipole interaction with the opposite chirality, the allowed CP asymmetry A_{asy} for each of the $B \to K\pi$ decay are similar as the above examples. The results are shown in Fig.(8).

A particular interesting case is that for $B^- \to \bar{K}^0\pi^-$. The SM predicts that $|A_{sym}(B^- \to \bar{K}^0\pi^-)| < 1\%$. In the [7], considering all kinds of uncertainty, $|A_{sym}(B^- \to \bar{K}^0\pi^-)|$ is still not larger than 2%. With SUSY dipole interaction, $|A_{asy}(B^- \to \bar{K}^0\pi^-)|$ can be as large as 10%. Observation of CP violation in $B^- \to \bar{K}^0\pi^-$ significantly large than SM prediction is an indication of new physics.

V. CONCLUSIONS

In this paper we have studied the contributions of SUSY gluonic dipole interaction to $B \to K\pi$ decays. We found that SUSY gluonic dipole interactions can affect $B \to K\pi$ significantly. QCD improved factorization calculation is lower than the current experimental branching ratio for $B^0 \to K^0\pi^0$. If experimental data will be further confirmed, this is an indication of new physics. We found that SUSY dipole interactions can improve the situation. All four measured $B \to K\pi$ decays can be in agreement with theoretical calculations with SUSY gluonic dipole interactions and at the same time satisfy constraint from $B \to X_s\gamma$.

Present data from $B \to K\pi$ can also further constrain the allowed parameter space for SUSY gluonic dipole interactions. A large portion of the parameter space allowed by $B \to X_s\gamma$ are excluded by $B \to K\pi$ data. SUSY gluonic dipole interaction coefficients r_g and δ are constrained to be into two narrow regions for both types of dipole chiralities.

Constraints from $B \to X_s\gamma$ and $B \to K\pi$ still allow large new CP violating phases in the

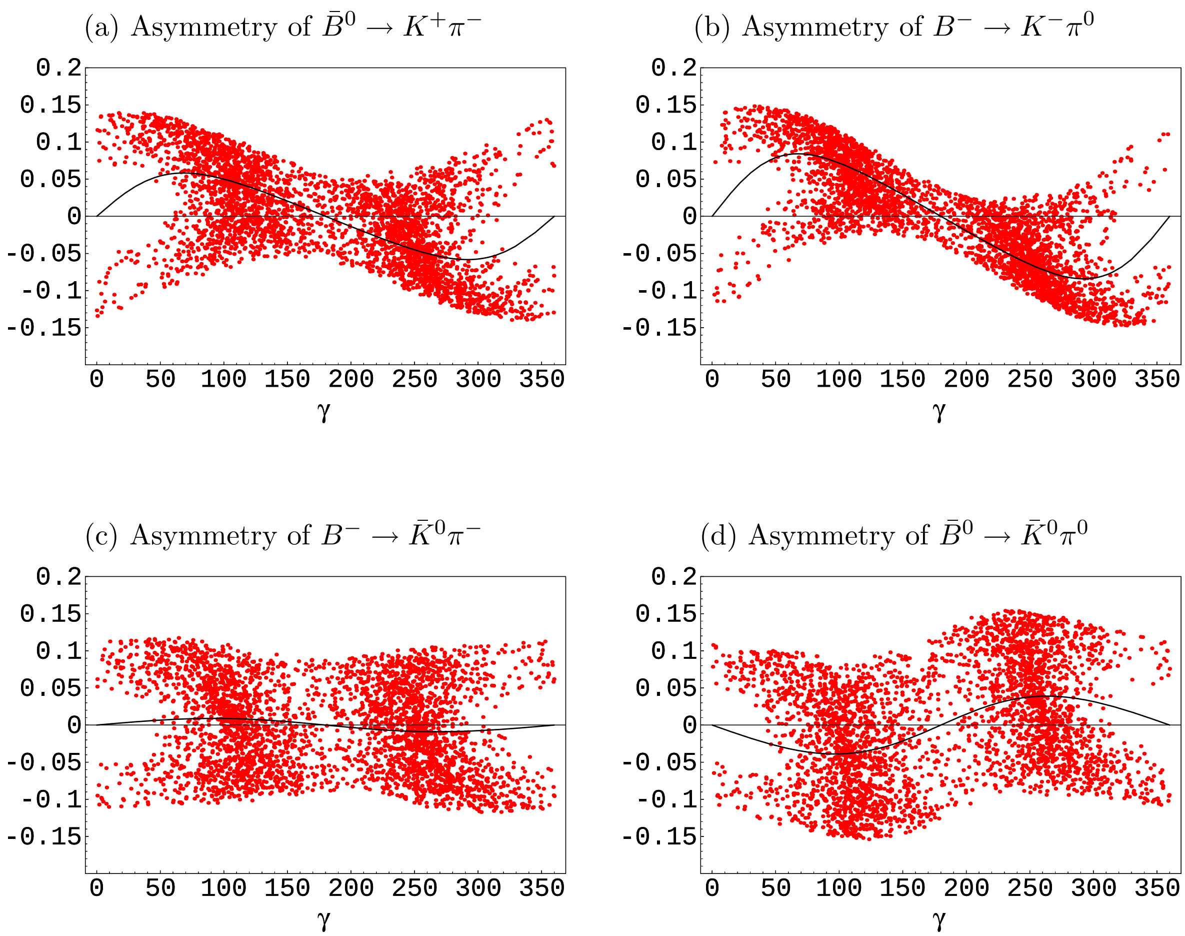

FIG. 7. The CP asymmetry A_{asy} for $B \to K\pi$ decays with SUSY gluonic dipole interaction having the same chirality as the SM one. The solid curves are SM predictions and the dotted regions are predictions with SUSY gluonic dipole interaction.

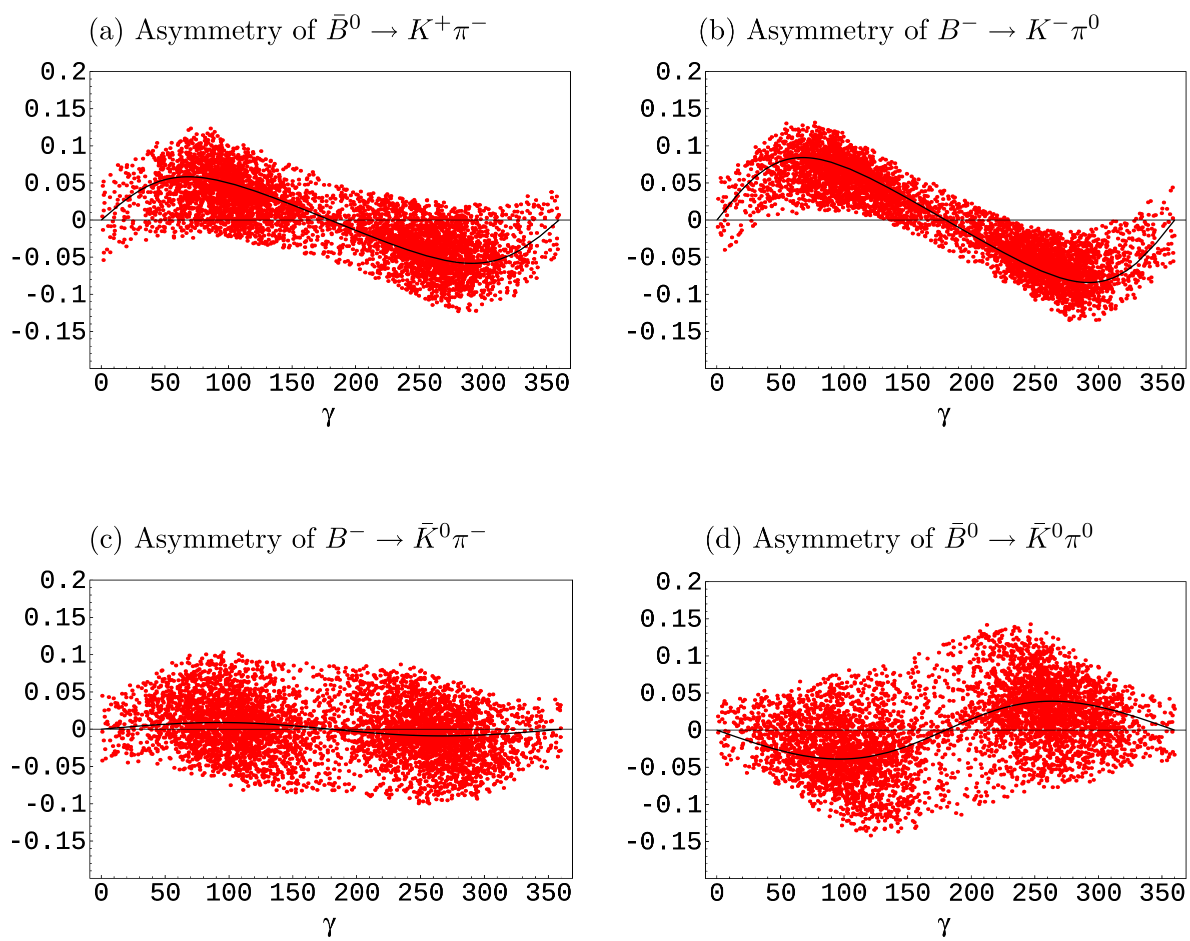

FIG. 8. The CP asymmetry A_{asy} for $B \to K\pi$ decays with SUSY gluonic dipole interaction having the opposite chirality as the SM one. The solid curves are SM predictions and the dotted regions are predictions with SUSY gluonic dipole interactions.

SUSY gluonic dipole interactions. This allows very different predictions for CP asymmetries for $B \to K\pi$ decays. In particular, with SUSY gluonic dipole interactions the value of CP asymmetry for $B^- \to \bar{K}^0\pi^-$ can be as large as 10% which is much larger than the SM prediction of less than 1%. This can provide an important test of new physics beyond the SM. CP violation in other modes can also be larger than the SM predictions.

Finally we would like to comment that with SUSY gluonic dipole contributions, some relations predicted in the SM may be violated. One such example is the rate difference, defined as $\Delta = \bar{\Gamma} - \Gamma$. In the Standard Model, one has $\Delta(\bar{B}^0 \to K^-\pi^+) = -(f_k/f_\pi)^2\Delta(\bar{B}^0 \to \pi^+\pi^-)$ [17],if kaon and π have the same wave function distribution amplitude.With different distribution amplitudes the relation is violated at 10% level. In obtaining this an important property of the CKM matrix element $Im(V_{ub}V^*_{us}V^*_{tb}V_{ts}) = -Im(V_{ud}V^*_{ud}V^*_{tb}V_{td})$ has been used. Since the SUSY contributions are proportional to $\delta^{bs}_{LR,RL}$ for $B \to K\pi$ decays, and proportional to $\delta^{bd}_{LR,RL}$ for $B \to \pi\pi$ decays. In general $\delta^{bs}_{LR,RL}$ and $\delta^{bd}_{LR,RL}$ are not related, the contributions from SUSY will break the relation $\Delta(\bar{B}^0 \to K^-\pi^+) = -(f_K/f_\pi)^2\Delta(\bar{B}^0 \to \pi^+\pi^-)$. We have checked with numerical calculations and found that indeed when SUSY contributions are included there are regions of parameter space where the relation mentioned is badly violated. Experimental measurements of these rate differences can also serve as tests of new physics beyond the Standard Model.

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